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CONVOLUTION APPROACH TO THE πNN SYSTEM

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ABSTRACT

The unitary $NN-\pi NN$ model contains a serious theoretical flaw: unitarity is obtained at the price of having to use an effective πNN coupling constant that is smaller than the experimental one. This is but one aspect of a more general renormalization problem whose origin lies in the truncation of Hilbert space used to derive the equations. Here we present a new theoretical approach to the πNN problem where unitary equations are obtained without having to truncate Hilbert space. Indeed, the only approximation made is the neglect of connected three-body forces. As all possible dressings of one-particle propagators and vertices are retained in our model, we overcome the renormalization problems inherent in previous πNN theories. The key element of our derivation is the use of convolution integrals that have enabled us to sum all the possible disconnected time-ordered graphs. We also discuss how the convolution method can be extended to sum all the time orderings of a connected graph. This has enabled us to calculate the fully dressed NN one pion exchange potential. We show how such a calculation can be used to estimate the size of the connected three-body forces neglected in the new πNN equations. Early indications are that such forces may be negligible.

INTRODUCTION

Eighteen months ago, at the International Few-Body Conference in Adelaide, one of us (B.B.) summarized the status of our understanding of the $NN-\pi NN$ system as follows [1], "Despite the almost thirty years of effort, we are forced to the conclusion that all current methods have serious deficiencies and that a radical new approach may be needed before this outstanding problem is finally resolved". In particular, it was demonstrated that the so called "unitary $NN-\pi NN$ model" [2-6], which had been the state of the art description for more than 10 years, is theoretically flawed. It is therefore especially pleasing to be able to present, to this conference, just such a radically new approach to the πNN system, involving convolution integrals, which resolves the outstanding theoretical

Figure 1: Allowed dressing in the unitary $NN-\pi NN$ model, with associated Z renormalization factors. (a) πN nucleon pole graph, (b) NN OPE graph.

problems of the unitary $NN-\pi NN$ model, and which, at this stage, promises to effectively take into account all the diagrams of time-ordered perturbation theory.[1]

As it still may not be well recognized that the unitary $NN-\pi NN$ model is flawed, let us first briefly repeat here the nature of the theoretical error inherent in the model, and how this error can lead to large inaccuracies in predictions of observables.

The origin of the problem lies in the truncation of Hilbert space used to derive the $NN-\pi NN$ equations. Truncation, in this case, means that explicit states with more than one pion are forbidden (certain multi-pion states are, however, included implicitly through the use of two-body, energy-independent potentials). This truncation has serious consequences for the renormalization of both the two-nucleon propagator and the πNN vertex. In Fig. 1(a) we show the πN nucleon pole diagram where the intermediate state nucleon is dressed by one-pion loops; however, the initial and final state nucleons do not include dressing since two-pion states are neglected in the truncation. Since close to the nucleon pole the dressed one-nucleon propagator is of the form $g(E) \sim Z/(E-m)$, where Z is the residue at the pole, Fig. 1(a) illustrates how each πNN vertex $f(E)$ gets effectively renormalized by a factor of $Z^{1/2}$. Thus $f_{\pi NN} = Z^{1/2} f(m)$ is essentially the πNN coupling constant, and this fact is used to fix the strength parameter in the form factor $f(E)$. With all other parameters of $f(E)$ fixed to reproduce experimental πN phase shifts, this form factor then enters the unitary $NN-\pi NN$ equations as an input. As illustrated in Fig. 1(b), when the NN one pion exchange (OPE) amplitude is calculated in the unitary $NN-\pi NN$ model, the initial and final nucleons are dressed by pions and consequently each external nucleon obtains a renormalization factor of $\tilde{Z}^{1/2}$. The first renormalization problem is the fact that $\tilde{Z} \neq Z$. This arises because two nucleons cannot be dressed at the same time in the truncated Hilbert space; thus, each nucleon in a two-nucleon state cannot obtain its full dressing. This, however, may not be such a serious problem since, in practice, the difference between Z and $\tilde{Z}$ turns out to be quite small. The serious problem, instead, is the size of the effective πNN coupling constant in the $NN-\pi NN$ equations. Taking $Z \approx \tilde{Z}$, Fig. 1(b) illustrates that each vertex gets renormalized by a factor of Z, so that the effective πNN coupling constant here becomes $Zf(m)$; this is a factor $Z^{1/2}$ times the physical coupling constant. With Z being typically between 0.6 and 0.8, we come to the disturbing conclusion that the effective πNN coupling constant in the $NN-\pi NN$ equations is smaller than the one used in constructing the input. This observation helps explain why one typically obtains much too small $pp \to \pi^+ d$ cross sections using this model [8-11].

These important observations about the renormalization problem in the unitary $NN-\pi NN$ model were already made in 1985 by Sauer *et al.* [12], yet they seem to have gone largely unnoticed. Perhaps this is partly because one could find less "fatal" reasons for the low cross sections; for example, it was legitimately argued that off-shell effects and the lack of a "backward-going pion" in the $NN \to N\Delta$ amplitude can lead to the underestimation of $pp \to \pi^+ d$ cross sections [10]. However, with the advent of calculations

[1] We note that progress has also been made in the four-dimensional relativistic sector where recently derived covariant equations for the πNN system resolve overcounting and undercounting problems of previous formulations [7]. In this review talk, however, we shall limit the discussion to the three-dimensional approaches of time-ordered perturbation theory.

where the nucleon and Δ are treated on an equal footing in the $NN-\pi NN$ model (the so-called $BB-\pi BB$ equations [1]), the effective πNN coupling constant is lowered by yet a further factor of $Z^{1/2}$ in the most important $NN \to N\Delta$ amplitude, and it has become very apparent that the renormalization problem is indeed fatal to this type of approach to the πNN system.

It may seem that one can fix the renormalization problem "by hand" by strategically including extra $Z^{1/2}$ factors in either πNN propagators or πNN form factors, or both. But it soon becomes apparent that there is no easy way of doing this without destroying the three-body unitarity of the equations.

Here we report on a completely different formulation of the πNN problem where unitary equations are obtained without having to truncate the Hilbert space to some maximum number of pions. Consequently, all possible dressings of one-particle propagators and vertices are retained in our model. In this way we overcome the renormalization problems discussed above. The key element of our derivation is the use of convolution integrals that have enabled us to sum all the possible disconnected time-ordered graphs [13]. Indeed, we basically make only one approximation in deriving the πNN convolution equations, namely, we neglect connected three-body forces.

As the details of this new approach have already been published [14], here we shall give a shortened derivation, retaining only the essential steps. A substantial part of our presentation involves an extension of the convolution idea to connected diagrams. In particular, we show how any diagram, connected or disconnected, can be calculated so that all possible dressing contributions are included. We then use the derived formalism to calculate the NN OPE potential where the nucleons are fully dressed. Not only does this calculation indicate that it may be important to include dressing into the popular one boson exchange models of the NN interaction, but it also provides for us a first check on the size of the neglected three-body forces. We find that, in this case, the three-body connected force contribution is negligible.

THE CONVOLUTION πNN EQUATIONS

We consider a time-ordered perturbation theory of nucleons and pions described by a Hamiltonian H. The exact form of H need not be specified. The Green function for the $\pi NN \to \pi NN$ process is thus defined by

$$\langle \mathbf{p}_1', \mathbf{p}_2', \mathbf{p}_3' | G(E) | \mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3 \rangle = \langle \mathbf{p}_1', \mathbf{p}_2', \mathbf{p}_3' | \frac{1}{E^+ - H} | \mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3 \rangle \tag{1}$$

where $\mathbf{p}_\alpha$ ($\mathbf{p}_\alpha'$) denote initial (final) momenta; here, as below, $\alpha = 1, 2$ label the two nucleons while $\alpha = 3$ denotes the pion. Note that we suppress spin-isospin labels in order to save on notation, and it is assumed that the nucleons are distinguishable since antisymmetrization can be carried out at the end.

We define the $\pi NN \to \pi NN$ t-matrix $T(E)$ by the equation

$$G(E) = G_0(E) + G_0(E) T(E) G_0(E) \tag{2}$$

where $G_0(E)$ is the free πNN propagator with all particles fully dressed (i.e. it is the fully disconnected part of $G(E)$). Note that in this section symbols G and g shall represent operators, while in the rest of the paper they are usually numbers; this should, in any case, be clear from the context. Eq. (2) is thus a relation between operators acting in the momentum space of one pion and two nucleons. In a previous work [13], we have shown how $G_0(E)$ can be expressed in terms of the individual pion and nucleon propagators by a convolution integral. In particular, if we write

$$\langle \mathbf{p}_\alpha' | g_\alpha(E) | \mathbf{p}_\alpha \rangle = \langle \mathbf{p}_\alpha' | \frac{1}{E^+ - H} | \mathbf{p}_\alpha \rangle \tag{3}$$

Figure 2: Diagrammatic representation of potentials (a) V_1, (b) V_3, and (c) V_4. The open circles represent all possible graphs excluding those that lead to πNN intermediate states. Potentials V_2 and V_5 are obtained by interchanging the two nucleons.

for the dressed nucleon and pion propagators, then $G_0(E)$ is given by

$$G_0(E) = \left(-\frac{1}{2\pi i}\right)^2 \int_{-\infty}^{\infty} dz_1 dz_2 \, g_1(E - z_1) g_2(z_1 - z_2) g_3(z_2). \tag{4}$$

Here it is understood that each single-particle propagator is an operator acting in its own Hilbert space.

With $T(E)$ being defined as in Eq. (2), we can write

$$T(E) = V(E) + V(E) G_0(E) T(E) \tag{5}$$

where the potential $V(E)$ is the sum of all possible πNN- irreducible graphs excluding those consisting of fully disconnected πNN states. At this point we shall neglect the connected diagrams of potential $V(E)$; these correspond to connected three-body forces and will be considered in the next section. It should be stressed that the connected diagrams of V are the only diagrams that are neglected in this model.

For the following discussion we follow the labelling convention where amplitudes that involve the pion interacting with nucleon i are labelled by subscript i, while the NN interaction with a pion spectator is labelled by subscript 3. Further, the indices i, j, are reserved for nucleons 1 and 2 while the Greek indices α, β, γ go from 1 to 3 unless otherwise indicated.

Of special relevance to the following discussion is the work of Stelbovics and Stingl (SS) [15], who investigated the disconnectedness problem in the πNN system, and then applied it to a model where Hilbert space is chopped at the two-pion level. As discussed by SS, all disconnected $3 \to 3$ diagrams belong to one of five classes of disconnectedness, denoted by δ_α, characterized by an appropriate momentum-space δ-function. Three of these classes, δ_1, δ_2, δ_3, have two of the particles interacting, the third being spectator. The class δ_4 has the pion in initial state being absorbed by nucleon 2 with the final state pion being emitted by nucleon 1. Class δ_5 is the same as δ_4 but with nucleons 1 and 2 interchanged.

According to this classification, the potential $V(E)$ can be written as the sum

$$V(E) = \sum_{\alpha=1}^{5} V_\alpha(E) \tag{6}$$

where $V_\alpha(E)$ consists of diagrams of disconnectedness δ_α. The potentials $V_\alpha(E)$ are represented diagrammatically in Fig. 2. It is worth mentioning that one of the contributions to the potential of Fig. 2(c) is the so called Jennings term which has been shown to be important for calculations of the tensor polarization t_{20} in πd scattering [16].

As it stands, Eq. (5) cannot be used directly for calculations as its kernel is disconnected. To derive equations with a compact kernel we proceed by analogy with the case of Faddeev equations and eliminate the potentials $V_\alpha(E)$ in Eq. (5) in favour of completely summed contributions of disconnectedness δ_α. Let us therefore denote by $\tilde{w}_\alpha$ the Green

function consisting of the set of *all* diagrams, reducible and irreducible, belonging to the disconnectedness class δ_α. We can then define the corresponding amplitudes w_α by

$$\tilde{w}_\alpha = G_0 w_\alpha G_0. \tag{7}$$

As $\sum_\alpha w_\alpha$ is the disconnected part of T, one can use Eq. (5) to equate the disconnected parts of type δ_α, and thus obtain relations between the V_α and the w_α. These relations can then be used to reexpress Eq. (5) as [15]

$$T = \sum_{\alpha=1}^{5} T_\alpha \tag{8}$$

$$T_\alpha = w_\alpha + \sum_{\beta=1}^{5} w_\alpha \kappa_{\alpha\beta} G_0 T_\beta \tag{9}$$

where $\kappa_{\alpha\beta}$ is a 5×5 matrix with elements $\kappa_{11} = \kappa_{22} = \kappa_{33} = \kappa_{51} = \kappa_{42} = \kappa_{14} = \kappa_{25} = \kappa_{45} = \kappa_{54} = 0$, all other elements being equal to 1. The first iteration of Eq. (9) then gives us a compact kernel.

As our equations and the ones of SS share the same disconnectedness structure, it is especially interesting to compare the actual models used. In the case of the SS model, the potentials V_α were specified by graphs of the lowest possible order. It then turned out that their w_α, needed to specify the kernel of Eq. (9), are not directly related to usual subsystem t-matrices, and must therefore be specified by solving very complicated integral equations [17]. By contrast, we take the most complicated possible choice for the V_α, namely the set of all possible contributing diagrams of field theory. The remarkable thing is, that it is just this maximally complicated choice that enables us to express the needed w_α as simple convolutions of dressed subsystem amplitudes (the usual two-body t-matrices t_α, the πNN vertex f, and the dressed one-nucleon propagator g) [13]. As these convolution integrals are written for Green function quantities, we shall utilize a "tilde" notation, as in Eq. (7), to label amplitudes with additional initial and final-state propagators. Thus for the πN t-matrix we define

$$\tilde{t}_i(E) = g_{\pi N_i}(E) t_i(E) g_{\pi N_i}(E) \tag{10}$$

where $g_{\pi N}$ is the dressed πN propagator. We note further that $\tilde{t}_i$ is the connected part of the full πN_i Green function, i.e.

$$\langle \mathbf{p}_i', \mathbf{p}_3' | \tilde{t}_i(E) | \mathbf{p}_i, \mathbf{p}_3 \rangle = \langle \mathbf{p}_i', \mathbf{p}_3 | \frac{1}{E^+ - H} | \mathbf{p}_i, \mathbf{p}_3 \rangle_c. \tag{11}$$

In a similar way we can define $\tilde{t}_3$. With these definitions, w_α can be written directly in terms of convolutions of $\tilde{t}_\alpha$ and the spectator particle propagators g_α. Introducing the short-hand notation

$$c = a \otimes b \tag{12}$$

to mean the convolution integral

$$c(E) = -\frac{1}{2\pi i} \int_{-\infty}^{\infty} dz\, a(E - z) b(z), \tag{13}$$

we have that $(i \neq j)$

$$\tilde{w}_i = \tilde{t}_i \otimes g_j \quad ; \quad \tilde{w}_3 = \tilde{t}_3 \otimes g_3. \tag{14}$$

It should be noted that $t_i(E)$ is the full πN t-matrix and can therefore be written in terms of nucleon pole and non-pole parts:

$$t_i(E) = f_i(E) g_i(E) \bar{f}_i(E) + t_i^b(E) \tag{15}$$

where $t_i^b(E)$ is the non-pole part, $f_i(E)$ is the dressed vertex for $N \to \pi N$, and $\bar{f}_i(E)$ is the dressed vertex for $\pi N \to N$. These vertices are explicitly given in terms of $\tilde{f}_i$ and $\bar{\tilde{f}}_i$:

$$\tilde{f}_i = g_{\pi N_i} f_i g_i \quad ; \quad \bar{\tilde{f}}_i = g_i \bar{f}_i g_{\pi N_i} \tag{16}$$

where

$$\langle \mathbf{p}_i', \mathbf{p}_3' | \tilde{f}_i(E) | \mathbf{p}_i \rangle = \langle \mathbf{p}_i', \mathbf{p}_3' | \frac{1}{E^+ - H} | \mathbf{p}_i \rangle \tag{17}$$

$$\langle \mathbf{p}_i' | \bar{\tilde{f}}_i(E) | \mathbf{p}_i, \mathbf{p}_3 \rangle = \langle \mathbf{p}_i' | \frac{1}{E^+ - H} | \mathbf{p}_i, \mathbf{p}_3 \rangle. \tag{18}$$

In a similar way, we can also express w_4 and w_5 directly in terms of the dressed πNN vertex using

$$\tilde{w}_4 = \tilde{f}_1 \otimes \bar{\tilde{f}}_2 \quad ; \quad \tilde{w}_5 = \tilde{f}_2 \otimes \bar{\tilde{f}}_1. \tag{19}$$

As the NN channel is hidden in the input terms w_α, the relation of Eq. (9) to the full NN amplitude is not clear. Neither is it apparent if in the process of iteration, the intermediate NN propagators will obtain their full dressing. For these reasons, we would like to recast Eq. (9) into a form that explicitly exposes the NN channel.

For this purpose, we write Eq. (9) in an AGS form [15]

$$U_{\alpha\beta} = \kappa_{\alpha\beta} G_0^{-1} + \sum_{\gamma=1}^{5} \kappa_{\alpha\gamma} w_\gamma G_0 U_{\gamma\beta}. \tag{20}$$

We note, however, that in the κ-matrix, rows 1 and 5 are identical, as are rows 2 and 4, columns 1 and 4, and columns 2 and 5. This means that

$$U_{\alpha 4} = U_{\alpha 1} \quad ; \quad U_{\alpha 5} = U_{\alpha 2} \quad ; \quad U_{4\beta} = U_{2\beta} \quad ; \quad U_{5\beta} = U_{1\beta}. \tag{21}$$

Thus although Eq. (20) is a set of 5×5 coupled equations, it can be reduced to a set of 3×3 equations.

We recall that the terms $\tilde{w}_i$ $(i = 1, 2)$ are defined as the set of all possible disconnected graphs for pion scattering on nucleon i. In order to expose two-nucleon states, we define the Green function $\tilde{w}_i^P$ as the set of all two-nucleon reducible graphs belonging to $\tilde{w}_i$, and write for the corresponding amplitudes

$$w_i = w_i^0 + w_i^P \tag{22}$$

where w_i^0 is two-nucleon irreducible. Since we consider all possible contributions, it is clear that one can write

$$w_i^P = F_i G_{NN} \bar{F}_i \tag{23}$$

where G_{NN} is the fully dressed two-nucleon propagator, F_i and $\bar{F}_i$, are the fully dressed πNN vertices in the two-nucleon sector. Again we can write these in terms of the one-nucleon propagator g_i and πNN vertices f_i and $\bar{f}_i$ through the convolution expressions $(i \neq j)$

$$G_{NN} = g_1 \otimes g_2, \tag{24}$$

$$G_0 F_i G_{NN} = \tilde{f}_i \otimes g_j, \tag{25}$$

$$G_{NN} \bar{F}_i G_0 = \bar{\tilde{f}}_i \otimes g_j. \tag{26}$$

In the same way we can separate out the two-nucleon reducible contributions from w_4 and w_5, and with the further convention that $w_3^P = 0$, we can write generally

$$w_\alpha = w_\alpha^0 + w_\alpha^P. \tag{27}$$

Clearly

$$w_4^P = F_1 G_{NN} \bar{F}_2 \quad ; \quad w_5^P = F_2 G_{NN} \bar{F}_1. \tag{28}$$

With these definitions we can define the amplitudes $U^0_{\alpha\beta}$ corresponding to all possible two-nucleon irreducible contributions, namely

$$U^0_{\alpha\beta} = \kappa_{\alpha\beta} G_0^{-1} + \sum_{\gamma=1}^{5} \kappa_{\alpha\gamma} w^0_\gamma G_0 U^0_{\gamma\beta}. \tag{29}$$

From Eq. (20), Eq. (27) and Eq. (29), it follows that

$$U_{\alpha\beta} = U^0_{\alpha\beta} + \sum_{\gamma=1}^{5} U^0_{\alpha\gamma} G_0 w^P_\gamma G_0 U_{\gamma\beta}. \tag{30}$$

Using the explicit forms for w^P_γ given in Eq. (23) and Eqs. (28), and making use of Eq. (21) to eliminate channels 4 and 5, we obtain for the last term in Eq. (30)

$$\sum_{\gamma=1}^{5} U^0_{\alpha\gamma} G_0 w^P_\gamma G_0 U_{\gamma\beta} = \left(U^0_{\alpha 1} G_0 F_1 + U^0_{\alpha 2} G_0 F_2\right) G_{NN} \left(\bar{F}_1 G_0 U_{1\beta} + \bar{F}_2 G_0 U_{2\beta}\right). \tag{31}$$

Substituting into Eq. (30) leads directly to the equation for nucleon-nucleon scattering

$$T_{NN}(E) = V_{NN}(E) + V_{NN}(E) G_{NN}(E) T_{NN}(E) \tag{32}$$

where

$$V_{NN}(E) = \sum_{ij} \bar{F}_i G_0 U^0_{ij} G_0 F_j \tag{33}$$

$$T_{NN}(E) = \sum_{ij} \bar{F}_i G_0 U_{ij} G_0 F_j. \tag{34}$$

This derivation shows explicitly that the NN propagator, which we did not have as an explicit input in Eq. (20), is indeed the fully dressed G_{NN} which is given in terms of the convolution integral of Eq. (24). Although it may be clear that the amplitude U_{ij} is made up of fully dressed component amplitudes, it is noteworthy that the connection to the NN scattering amplitude is via the vertex functions F_i and $\bar{F}_i$ which themselves have all possible dressings.

A feature of our formulation is that the input to our derived equations, Eq. (9) or equivalently Eq. (20), consists just of usual two-body subsystem off-shell amplitudes. This is an important aspect for practical calculations and was not the case for the formally similar equations of SS. On the other hand, due to the terms w^0_4 and w^0_5, the kernel of the equations nevertheless is not of the standard type where all interactions are of two-body type. We shall now show how one can rewrite the equations to have standard pair-like interactions in the kernel but at the expense of introducing an extra dimension in the NN sector.

We start by writing an alternative decomposition of w_α to that given in Eq. (27). For w_i we convolute t_i, as given in Eq. (15), with a dressed spectator nucleon; that is, the decomposition

$$w_i = \bar{w}^0_i + \bar{w}^P_i \tag{35}$$

will be defined via the convolutions

$$G_0 \bar{w}^P_i G_0 = (g_{\pi N} f g \bar{f} g_{\pi N})_i \otimes g_j \tag{36}$$

$$G_0 \bar{w}^0_i G_0 = (g_{\pi N} t^b g_{\pi N})_i \otimes g_j. \tag{37}$$

We can now write $w_\alpha = \bar{w}^0_\alpha + \bar{w}^P_\alpha$ generally by defining $\bar{w}^P_3 = 0$, $\bar{w}^P_4 = w_4$, $\bar{w}^P_5 = w_5$, $\bar{w}^0_3 = w_3$, $\bar{w}^0_4 = 0$, and $\bar{w}^0_5 = 0$. Analogous to Eq. (36) we have that

$$G_0 \bar{w}^P_4 G_0 = (g_{\pi N} f g)_1 \otimes (g \bar{f} g_{\pi N})_2, \tag{38}$$
$$G_0 \bar{w}^P_5 G_0 = (g_{\pi N} f g)_2 \otimes (g \bar{f} g_{\pi N})_1, \tag{39}$$

which are just more explicit versions of Eq. (19).

Introducing this decomposition of w_α into Eq. (20), leads us to define, by analogy to Eq. (29),

$$\bar{U}^0_{\alpha\beta} = \kappa_{\alpha\beta} G_0^{-1} + \sum_{\gamma=1}^{5} \kappa_{\alpha\gamma} \bar{w}^0_\gamma G_0 \bar{U}^0_{\gamma\beta}. \tag{40}$$

Analogous expressions to Eq. (30) and Eq. (31) are also easily written. Using a short-hand notation, illustrated generally by

$$(ab)_{1z} = a_1(E-z) b_1(E-z) \quad ; \quad (ab)_{2z} = a_2(z) b_2(z), \tag{41}$$

we are then led to define a new kind of NN potential and t-matrix depending on extra variables:

$$V_{NN}(z', z; E) = \sum_{ij} (\bar{f} g_{\pi N})_{iz'} \bar{U}^0_{ij} (g_{\pi N} f)_{jz} \tag{42}$$
$$T_{NN}(z', z; E) = \sum_{ij} (\bar{f} g_{\pi N})_{iz'} U_{ij} (g_{\pi N} f)_{jz}. \tag{43}$$

In this way the integral equation for NN scattering acquires an extra dimension,

$$T_{NN}(z', z; E) = V_{NN}(z', z; E) - \frac{1}{2\pi i} \int_{-\infty}^{\infty} dz''\, V_{NN}(z', z''; E) g_1(E - z'') g_2(z'') T_{NN}(z'', z; E). \tag{44}$$

How $T_{NN}(z', z; E)$ can be identified with the physical NN t-matrix, $T_{NN}(E)$, can be seen as follows. From Eq. (25), it is easy to show that in the case when the two initial-state nucleons are on-energy-shell, i.e. $\mathbf{p}_1^2/2m + \mathbf{p}_2^2/2m + 2m = E$, then

$$g_{\pi N_i}(\mathbf{p}_i^2/2m + m) f_i(\mathbf{p}_i^2/2m + m) |\mathbf{p}_1 \mathbf{p}_2\rangle = G_0(E) F_i(E) |\mathbf{p}_1 \mathbf{p}_2\rangle \tag{45}$$

with a similar equation holding for $\bar{f}_i g_{\pi N_i}$. It can therefore be seen from Eq. (43) that if the initial and final nucleons are on-energy-shell, and if we set the z variables to the energies of the second nucleon, i.e. $z' = \mathbf{p}_2'^{\,2}/2m + m$, $z = \mathbf{p}_2^2/2m + m$, then

$$\langle \mathbf{p}_1' \mathbf{p}_2' | T_{NN}(z', z; E) | \mathbf{p}_1 \mathbf{p}_2 \rangle = \langle \mathbf{p}_1' \mathbf{p}_2' | T_{NN}(E) | \mathbf{p}_1 \mathbf{p}_2 \rangle \tag{46}$$

where $T_{NN}(E)$ is given in Eq. (34). In this sense $T_{NN}(z', z; E)$ can be considered as an off-shell scattering amplitude with additional energy-like variables z and z'.

Our final result for the description of the πNN system consists of the coupled equations (40) and (44). The equation for $\bar{U}^0$ is just the Faddeev equation for the πNN system with no absorption. In momentum space it is a 6-dimensional integral equation and involves pair-interactions π-N and N-N.

The equation for NN scattering, Eq. (44), in momentum space, is a 4-dimensional integral equation. In this sense it is similar to the Bethe-Salpeter equation but with particular forms for the nucleon propagator and NN potential. In the case of Eq. (44), the nucleon propagator contains only positive energy states while the one-pion exchange potential, for example, has πNN vertex functions that depend only on one energy variable

- in the Bethe-Salpeter case they depend on two energy variables. It is interesting to note that by introducing a fourth dimension into the NN sector, we take into account the NN irreducible diagrams in w_4 and w_5 while at the same time having only pair-like interactions in the kernel for $\bar{U}^0$.

We conclude by noting that our derivation is model independent. Thus, for example, there was no need to specify either how pions actually couple to nucleons, or what model is used for dressing the pion. We also note that our expressions for the coupled πNN equations, Eq. (40) and Eq. (44), or alternatively the coupled equations (29) and (34), express the main features of the physics. Thus, for example, we can easily check that our formulation gives consistent renormalization, simply by substituting the pole term $Z/(E^+ - m)$ wherever the single-nucleon propagator $g(E)$ appears. The unitarity of our equations is also evident from their derivation. For practical calculations, however, either of the coupled set of equations can easily be cast into one set of coupled equations for the reactions $NN \to NN$, $\pi d \to \pi d$, and $NN \to \pi d$. In the case of Eq. (40) and Eq. (44), this would result in coupled equations that are of similar form to the well known unitary $NN - \pi NN$ equations [5], but with an extra dimension in the NN sector.

THE DRESSED *NN* OPE POTENTIAL

The essential feature of the πNN equations, presented above, is that they include all possible disconnected πNN-irreducible graphs. This is the first time that *all* such disconnected diagrams have been included, and has been achieved only after preliminary work showing that convolution integrals can sum all possible time orderings of *disconnected* graphs of time-ordered perturbation theory [13].

With all the disconnected diagrams included, the focus of attention turns naturally to the question of how large the neglected *connected* πNN-irreducible graphs are. This is not an easy question to answer without a systematic analysis of the whole range of such connected three-body force contributions. Here we shall take the first steps in such an analysis by examining the most basic ingredient of the NN force, namely, the NN one pion exchange potential.

Within the formalism of the πNN convolution equations, the OPE potential takes the form appearing in Eq. (33) with U^0_{ij} replaced by G_0^{-1} (the inhomogeneous term of Eq. (29)). It will be sufficient to examine just one time ordering of the exchanged pion, and in this case, the OPE potential of the πNN convolution equations is given by

$$V^{OPE}_{12} = \bar{F}_1 G_0 F_2. \tag{47}$$

The consequence of neglecting connected three-body forces becomes immediately apparent upon examination of the dressing contributions to this OPE potential. In Eq. (47), each of the terms $\bar{F}_1$, G_0, and F_2, is by itself completely dressed. Thus a simple example of a dressing contribution that is included in Eq. (47) is given by Fig. 3(a). On the other hand, the topologically similar graph of Fig. 3(b) is not included in the formalism of the previous section since it is of the form $\bar{F}_2 V^c F_1$ where V^c is the connected $\pi NN \to \pi NN$ three-body force.

In order to see the effect of neglecting dressings of the OPE potential like that of Fig. 3(b), it would be useful to have an expression for the fully dressed OPE potential. That is, we need the OPE graph where all possible dressing graphs involving a πNN vertex are retained (the pion itself, however, will be assumed to have no dressing contributions). As will be shown in the sections below, such an expression can indeed be easily derived by extending the convolution idea to connected diagrams. Here we just give the result for

Figure 3: (a) Example of a dressing diagram included in the convolution πNN equations. (b) Example of a dressing diagram involving a connected three-body force - such diagrams are not included in the convolution πNN equations.

the corresponding OPE Green function (in the c.m. system) with momentum conserving δ-functions removed:

$$\tilde{V}_{12}^{OPE}(\mathbf{p}',\mathbf{p};E) = \left(-\frac{1}{2\pi i}\right)^2 \int dz\, dz'\, g(z',\mathbf{p}')\bar{f}(\mathbf{p}',\mathbf{p},z',z)g(z,\mathbf{p})\frac{1}{z'-z-\omega_k} \\ g(E-z',-\mathbf{p}')f(-\mathbf{p}',-\mathbf{p},E-z',E-z)g(E-z,-\mathbf{p}). \quad (48)$$

Here ω_k is the energy of the exchanged pion, and $g(E,\mathbf{p})$ is the dressed nucleon propagator defined as the matrix element of Eq. (3), but with the momentum conserving δ-function removed, i.e.

$$\delta(\mathbf{p}-\mathbf{p}')g(E,\mathbf{p}) = \langle \mathbf{p}'|\frac{1}{E^+-H}|\mathbf{p}\rangle. \quad (49)$$

A novel feature of Eq. (48) is the need for a πNN vertex function $f(\mathbf{p}',\mathbf{p},E',E)$ that depends on two energy variables. By contrast, in the πNN convolution equations, as in most other formulations utilizing time-ordered perturbation theory, the dressed vertex depends only on one energy variable. The relation between one- and two-energy vertices will be given below.

The essential point here is that if one neglects connected three-body forces from Eq. (48), then one will obtain the OPE Green function of the convolution πNN equations, which numerically is given as a product of three convolution integrals:

$$\tilde{V}_{12}^{OPE}(\mathbf{p}',\mathbf{p};E) = \left(-\frac{1}{2\pi i}\right)^3 \left[\int dz\, g(z,\mathbf{p}')\bar{f}(\mathbf{p}',\mathbf{p},z)g(z,\mathbf{p})g(E-z,-\mathbf{p}')\right] \\ \left[\int dz\, g(z,\mathbf{p})g(E-\omega_k-z,-\mathbf{p}')\right]\left[\int dz\, g(z,-\mathbf{p}')f(-\mathbf{p}',-\mathbf{p},z)g(z,-\mathbf{p})g(E-z,\mathbf{p})\right]. \quad (50)$$

The fully off-shell OPE potential $V_{12}^{OPE}(\mathbf{p}',\mathbf{p};E)$ is related to the OPE Green function in the usual way:

$$V_{12}^{OPE}(\mathbf{p}',\mathbf{p};E) = G_{NN}^{-1}(E,\mathbf{p}',-\mathbf{p}')\tilde{V}_{12}^{OPE}(\mathbf{p}',\mathbf{p};E)G_{NN}^{-1}(E,\mathbf{p},-\mathbf{p}) \quad (51)$$

where G_{NN} is the dressed two-nucleon propagator given as in Eq. (24) . For the numerical comparison of Eqs. (48) and (50) we follow our previous work [13] and use the $M1$ πN interaction of Ref. [11]. We shall, in each case, calculate the half-off-shell potential $V_{12}^{OPE}(\mathbf{p}_0,\mathbf{p};E)$, defined by Eq. (51) in the limiting case $\mathbf{p}' \to \mathbf{p}_0$, where $\mathbf{p}_0$ is the on-shell momentum, i.e. $E = p_0^2/m + 2m$. Here we shall consider the simplified case where no dressing is included for the vertices of the exchanged pion; in this case, both the two-energy vertex $f(\mathbf{p}',\mathbf{p},E',E)$ of Eq. (48) and the single-energy vertex $f(\mathbf{p}',\mathbf{p},E)$ of Eq. (50), reduce down to the energy-independent bare vertex $f_0(\mathbf{p}',\mathbf{p})$. Note that the restriction to bare vertices is limited to the pion exchanged between the two nucleons, so that the dressed nucleon propagators g are totally unaffected by this simplification. The half-off-shell potential $V_{12}^{OPE}(\mathbf{p}_0,\mathbf{p};E)$ depends on three variables, the energy E, the magnitude p of the off-shell momentum, and the cosine of the angle between $\mathbf{p}_0$ and $\mathbf{p}$.

Figure 4: Comparison of the half-off-shell dressed NN OPE potential, $V_{12}^{OPE}(\mathbf{p}_0, \mathbf{p}; E)$ calculated with full dressing (solid curves), and within the convolution πNN model (long-dashed curves). The short-dashed curve is for the case where no dressing is included.

For the numerical comparison, we have examined the potential as a function of energy E for a large range of values of p and $x = \hat{\mathbf{p}}_0 \cdot \hat{\mathbf{p}}$. A typical result is shown in Fig. 4 where, in this case, we have set $p = .1p_0$ and $x = -0.8$. In this figure, the solid curve gives the OPE potential with full dressing, while the long-dashed curve is the result when connected three-body forces are neglected. Also shown is the standard OPE potential where no dressing at all is included (short-dashes). The first observation of note is the significant effect that dressing has on the standard OPE potential. This raises questions about the role such dressing may play in standard descriptions of the NN force. The second observation forms the main result of this section - it is the essential identity of the solid and long-dashed curves, indicating that the contribution of connected three-body forces to the OPE potential is negligible. Although strictly applying to the OPE potential with bare vertices, this is a very encouraging result that gives us hope that connected three-body forces may be small in general. If this is borne out in further studies, this would mean that, for the πNN system, the convolution πNN equations provide a way of effectively summing all possible diagrams of time-ordered perturbation theory.

DRESSING OF CONNECTED DIAGRAMS - METHOD I

In Ref. [13] we showed how a convolution integral can sum all possible relative time orderings of a *disconnected* diagram of time-ordered perturbation theory. This was then used to demonstrate how two disconnected nucleons can be dressed simultaneously. Here we show that the convolution idea can also be easily extended to *connected* diagrams. The final result shows that the sum of all topologically similar diagrams, connected or disconnected, differing only in the relative time ordering of their vertices, can be expressed through a convolution formula that effectively integrates out initial and final relative energies from the topologically equivalent Feynman diagram. One application of this generalized convolution formula is to the problem of including all possible dressings in connected diagrams.

The starting point of the following discussion is relativistic quantum field theory. Since Feynman diagrams contain, in some sense, all time orderings, it is not surprising that our goal will be to show how to extract the Green function of time-ordered perturbation theory from the Green function of relativistic quantum field theory. Although the procedure we'd like to follow is general, for presentation purposes we specifically consider the $NN \to NN$ process where the interaction is described by a Hamiltonian $H = H_0 + H_I$ involving meson

and baryon fields. The explicit form of H need not be specified.

The free fermion field at time $t = 0$ is denoted by $\psi(\mathbf{x})$. At time t, we define

$$\psi(\mathbf{x}, t) = e^{iH_0 t}\psi(\mathbf{x})e^{-iH_0 t} \tag{52}$$

$$\Psi(\mathbf{x}, t) = e^{iHt}\psi(\mathbf{x})e^{-iHt} \tag{53}$$

being the interaction picture and Heisenberg fields, respectively. The free fermion field $\psi(x) = \psi(\mathbf{x}, t)$ can then be written in terms of its Fourier decomposition as [18]

$$\psi(x) = \sum_s \int \frac{d^3p}{(2\pi)^{3/2}} \sqrt{\frac{m}{E_p}} [b(\mathbf{p}, s)u(\mathbf{p}, s)e^{-ip\cdot x} + d^\dagger(\mathbf{p}, s)v(\mathbf{p}, s)e^{ip\cdot x}]. \tag{54}$$

We consider the $NN \to NN$ process described by the coordinate space Green function

$$i\mathcal{G}(x, y; x', y') = \langle\langle 0|T\Psi(x)\Psi(y)\bar{\Psi}(x')\bar{\Psi}(y')|0\rangle\rangle, \tag{55}$$

which, because each Ψ is a four-component spinor, can be considered as a 16×16 matrix. Here $|0\rangle\rangle$ is the dressed vacuum, its relation to the bare vacuum $|0\rangle$ being given by

$$|0\rangle\rangle = \frac{1}{\langle\langle 0|0\rangle\rangle}[1 + \frac{1}{0^+ - H}H_I]|0\rangle. \tag{56}$$

The momentum space Green function is defined by

$$\mathcal{G}(p, q; p', q') = \int e^{i(x\cdot p + y\cdot q - x'\cdot p' - y'\cdot q')}\mathcal{G}(x, y; x', y')d^4x\, d^4y\, d^4x'\, d^4y', \tag{57}$$

which can also be expressed without the momentum conserving δ-function by defining the Green function $G(p, q; p', q')$:

$$\mathcal{G}(p, q; p', q') = (2\pi)^4\delta^4(p' + q' - p - q)G(p, q; p', q'). \tag{58}$$

We follow Logunov and Tavkhelidze [19] and consider the two-time Green function

$$\mathcal{G}(\mathbf{x}, \mathbf{y}, t; \mathbf{x}', \mathbf{y}', t') \equiv \mathcal{G}(x, y; x'y')\Big|_{\substack{x_0 = y_0 = t \\ x'_0 = y'_0 = t'}}. \tag{59}$$

The two-time Green function in momentum space is defined by

$$\tilde{\mathcal{G}}(\mathbf{p}, \mathbf{q}, E; \mathbf{p}', \mathbf{q}', E') \equiv \int e^{i(-\mathbf{x}\cdot\mathbf{p} - \mathbf{y}\cdot\mathbf{q} + \mathbf{x}'\cdot\mathbf{p}' + \mathbf{y}'\cdot\mathbf{q}' + tE - t'E')} \mathcal{G}(\mathbf{x}, \mathbf{y}, t; \mathbf{x}', \mathbf{y}', t')d^3x\, d^3y\, d^3x'\, d^3y'\, dt\, dt'. \tag{60}$$

A little algebra shows that

$$\tilde{\mathcal{G}}(\mathbf{p}, \mathbf{q}, E; \mathbf{p}', \mathbf{q}', E') = \frac{1}{(2\pi)^2}\int_{-\infty}^{\infty} d\omega \int_{-\infty}^{\infty} d\omega'\, \mathcal{G}(\mathbf{p}, E - \omega, \mathbf{q}, \omega; \mathbf{p}', E' - \omega', \mathbf{q}', \omega'). \tag{61}$$

This result was used by Logunov and Tavkhelidze as the starting point for their study of the two-time Green function. It also constitutes the solution to our problem as we now proceed to show.

It is useful to express $\tilde{\mathcal{G}}$ without momentum conserving δ-functions, thus we define the Green function $\tilde{G}$ by

$$\tilde{\mathcal{G}}(\mathbf{p}, \mathbf{q}, E; \mathbf{p}', \mathbf{q}', E') = (2\pi)^4\delta(\mathbf{p} + \mathbf{q} - \mathbf{p}' - \mathbf{q}')\delta(E - E')\tilde{G}(E, \mathbf{p}, \mathbf{q}; \mathbf{p}', \mathbf{q}'). \tag{62}$$

Removing the momentum conserving δ-functions from Eq. (61), gives

$$\tilde{G}(E,\mathbf{p},\mathbf{q};\mathbf{p}',\mathbf{q}') = \frac{1}{(2\pi)^2}\int_{-\infty}^{\infty} d\omega \int_{-\infty}^{\infty} d\omega'\, G(\mathbf{p},E-\omega,\mathbf{q},\omega;\mathbf{p}',E-\omega',\mathbf{q}',\omega'). \tag{63}$$

The rhs of Eq. (63) is a double convolution over initial and final relative energies of Feynman graphs. The goal, therefore, is to show how the lhs, $\tilde{G}(E,\mathbf{p},\mathbf{q};\mathbf{p}',\mathbf{q}')$, is related to the Green function of time-ordered perturbation theory. For this we go back to coordinate space and examine the two-time Green function in detail. By the definition of the time-ordered product in Eq. (55),

$$\begin{aligned} i\mathcal{G}(\mathbf{x},\mathbf{y},t;\mathbf{x}',\mathbf{y}',t') &= \theta(t-t')\langle\langle 0|\Psi(\mathbf{x},t)\Psi(\mathbf{y},t)\bar{\Psi}(\mathbf{x}',t')\bar{\Psi}(\mathbf{y}',t')|0\rangle\rangle \\ &+ \theta(t'-t)\langle\langle 0|\bar{\Psi}(\mathbf{x}',t')\bar{\Psi}(\mathbf{y}',t')\Psi(\mathbf{x},t)\Psi(\mathbf{y},t)|0\rangle\rangle. \end{aligned} \tag{64}$$

Using Eq. (53) in this equation, and taking the Fourier transform as in Eq. (60) gives the result

$$\begin{aligned} (2\pi)^3\delta(\mathbf{p}+\mathbf{q}-\mathbf{p}'-\mathbf{q}')\tilde{G}(E,\mathbf{p},\mathbf{q};\mathbf{p}',\mathbf{q}') &= \langle\langle 0|\psi(\mathbf{p})\psi(\mathbf{q})\frac{1}{E^+-H}\bar{\psi}(\mathbf{p}')\bar{\psi}(\mathbf{q}')|0\rangle\rangle \\ &- \langle\langle 0|\bar{\psi}(\mathbf{p}')\bar{\psi}(\mathbf{q}')\frac{1}{E^-+H}\psi(\mathbf{p})\psi(\mathbf{q})|0\rangle\rangle \end{aligned} \tag{65}$$

where we have used Eq. (62), the fact that

$$\int_{-\infty}^{\infty}\theta(t)e^{i\omega t}dt = \frac{i}{\omega+i\epsilon}, \tag{66}$$

and where we have introduced the momentum space fields

$$\psi(\mathbf{p}) \equiv \int d\mathbf{x}^3\, e^{-i\mathbf{x}\cdot\mathbf{p}}\psi(\mathbf{x}). \tag{67}$$

Using Eq. (54) at $t=0$ in Eq. (67), one obtains that

$$\psi(\mathbf{p}) = \sum_s \sqrt{\frac{(2\pi)^3 m}{E_p}}[b(\mathbf{p},s)u(\mathbf{p},s) + d^\dagger(-\mathbf{p},s)v(-\mathbf{p},s)]. \tag{68}$$

Eq. (65) is basically the result that we seek. It expresses $\tilde{G}$ in terms of two terms corresponding to the retarded and advanced parts of the Green function of Eq. (64); we shall correspondingly name the first and second terms on the rhs of Eq. (65) as retarded and advanced, respectively.

One may compare Eq. (65) with the Green function of time-ordered perturbation theory, which for the process in question is given by

$$(2\pi)^3\delta(\mathbf{p}+\mathbf{q}-\mathbf{p}'-\mathbf{q}')G_{\alpha\beta,\alpha'\beta'}(E,\mathbf{p},\mathbf{q};\mathbf{p}',\mathbf{q}') = \langle\mathbf{p},\alpha,\mathbf{q},\beta|\frac{1}{E^+-H}|\mathbf{p}',\alpha',\mathbf{q}',\beta'\rangle \tag{69}$$

where the spin components $\alpha,\beta,\alpha',\beta'$ are shown explicitly. Taking into account Eq. (68), we see that the retarded part of $\tilde{G}$ in Eq. (65) is very similar to the Green function of Eq. (69). This similarity suggests a simple transformation of Eq. (65) defined by the equation

$$G_{\alpha\beta,\alpha'\beta'} = \left[\frac{1}{(2\pi)^3 m}\sqrt{E_pE_q}\bar{u}(\mathbf{p},\alpha)\bar{u}(\mathbf{q},\beta)\right] G \left[u(\mathbf{p}',\alpha')u(\mathbf{q}',\beta')\frac{1}{(2\pi)^3 m}\sqrt{E_{p'}E_{q'}}\right] \tag{70}$$

where here $G = G(\mathbf{p}, \mathbf{q}, \mathbf{p}', \mathbf{q}')$ represents any appropriate Green function. Note that Eq. (70) transforms a 16×16 matrix in spinor space to a 4×4 matrix in spin space. Under this transformation, Eq. (65) becomes

$$(2\pi)^3\delta(\mathbf{p}+\mathbf{q}-\mathbf{p}'-\mathbf{q}')\tilde{G}_{\alpha\beta,\alpha'\beta'}(E,\mathbf{p},\mathbf{q};\mathbf{p}',\mathbf{q}') = \langle\langle 0|b(\mathbf{p},\alpha)b(\mathbf{q},\beta)\frac{1}{E^+-H}b^\dagger(\mathbf{p}',\alpha')b^\dagger(\mathbf{q}',\beta')|0\rangle\rangle - \langle\langle 0|b^\dagger(\mathbf{p}',\alpha')b^\dagger(\mathbf{q}',\beta')\frac{1}{E^-+H}b(\mathbf{p},\alpha)b(\mathbf{q},\beta)|0\rangle\rangle. \quad (71)$$

If $|0\rangle\rangle = |0\rangle$, the advanced term of Eq. (71) disappears, and the retarded term becomes identical with the Green function of time ordered perturbation theory, Eq. (69). Thus the only difference between the transformed two-time Green function of Eq. (71) and the one of time-ordered perturbation theory, is in the type of vacua used to take the matrix element: the transformed version of $\tilde{G}$ uses the dressed vacuum while standard time-ordered perturbation theory, as in Eq. (69), uses the bare vacuum.

In general, $|0\rangle\rangle \neq |0\rangle$, however, in the special case where the dressed and bare vacua are equal, Eqs. (69) and (71) become identical. In this case, Eq. (63), after the transformation of Eq. (70), provides us with a formula that expresses the full Green function of time-ordered perturbation theory in terms of a double convolution integral of the full Feynman Green function. This result can be easily generalized to hold for perturbation diagrams of a given order in the interaction, or even for perturbation diagrams of a given topology (the argument is similar to the one used in Ref. [13]). It is in this latter form that the method can give particularly useful formulas, like that of Eq. (48), where all possible time-orderings of a perturbation graph are summed by performing convolution integrals of one Feynman diagram.

A similar result to ours has lately been obtained by Phillips and Afnan [20] using a much more involved argument. In their case, however, they neglected antinucleons altogether in order to obtain the connection of Eq. (63) to time-ordered perturbation theory.[2]

We emphasize that all that is needed in our derivation is the condition $|0\rangle\rangle = |0\rangle$. For this to be true, it is sufficient to drop terms from the interaction Hamiltonian that connect vacuum to vacuum states, otherwise antinucleons can be retained. For example, in the simple case of interaction $H_I \sim \bar{\psi}\phi\psi$, expanding each field in terms of creation and annihilation operators gives eight terms; however, of these eight, only terms $b^\dagger a^\dagger d^\dagger$ and dab contribute to the dressing of the vacuum (in lowest order via the process $0 \to N\bar{N}\pi \to 0$), and just these terms can be dropped while still retaining other terms involving d and $d^\dagger$.

DRESSING OF CONNECTED DIAGRAMS - METHOD II

The above method using the two-time Green function can be used to derive Eq. (48) for the dressed NN OPE potential, but only in the case where vacuum dressing has been neglected. It turns out, however, that Eq. (48) is more general, holding for *any* Hamiltonian H, including ones involving vacuum dressing.

Here we derive Eq. (48) using a totally different method that is close in spirit to the one used to derive the convolution formula for disconnected diagrams [13]. This method

[2]This paper claims, incorrectly, to have shown that the convolution formula, derived by us in Ref. [13], is a special case of their two-time approach where antinucleons are neglected. If anything, just the opposite is the case: our convolution formula was derived using a method, akin to Method II, which does not need for its validity any approximations whatsoever. We also do not share these authors' criticism of three-dimensional approaches, even though we have ourselves already presented the first consistent four-dimensional approach [7].

involves the temporary introduction of different types of pions and nucleons into a time-ordered perturbation theory description, and provides a way to derive Eq. (48) without making any approximations. In this sense, this method is more general than the one based on the two-time Green function.

The central idea is to interpret the dressed NN OPE potential as consisting of two distinguishable nucleons N_1 and N_2, each being dressed by its own pion, π_1 and π_2 respectively, and in addition, exchanging a third type of pion π; of the three types of pion, only π can interact with both nucleons. We accordingly define $H_1 = H_0(1) + H_I(1)$ to be the Hamiltonian describing the $\pi_1 N_1$ system, and similarly $H_2 = H_0(2) + H_I(2)$ is the Hamiltonian describing the $\pi_2 N_2$ system. The Hamiltonian describing pion π and its interactions with the nucleons can be similarly written as $H^\pi = H_0^\pi + H_I^\pi(1) + H_I^\pi(2)$.

Thus, given any Hamiltonian $H = H_0 + H_I$ for which we would like to calculate the NN OPE potential, we can proceed by replacing H with the sum $H' = H_1 + H_2 + H^\pi$, where each individual Hamiltonian H_1, H_2, and H^π has free and interaction parts that are of the same form as H_0 and H_I, respectively, but each with its own individual fields replacing the corresponding ones of H.

Although it may be possible to provide a general formulation, it is more convenient to illustrate the procedure to follow by taking the usual model where the interaction is given by a three-point πNN vertex.

As we shall not explicitly need to use any details of the model for H_1 and H_2, we specify the model in terms of the Hamiltonian involving the pion π:

$$H_0^\pi = \int d\mathbf{k}\, \omega_k\, a_\pi^\dagger(\mathbf{k}) a_\pi(\mathbf{k}) \tag{72}$$

$$H_I^\pi = \int d\mathbf{k}\, a_\pi^\dagger(\mathbf{k}) J_N(\mathbf{k}) + H.c. \tag{73}$$

$$J_N(\mathbf{k}) = \int d\mathbf{p}\, d\mathbf{p}' \delta(\mathbf{p} + \mathbf{k} - \mathbf{p}') \frac{1}{\sqrt{\omega_k}} F_0(\mathbf{p}, \mathbf{p}') a_N^\dagger(\mathbf{p}) a_N(\mathbf{p}') \tag{74}$$

where N can be either N_1 or N_2, in which case H_I^π needs also to be labelled accordingly. Note the relations

$$[a_\pi(\mathbf{k}), H_0^\pi] = \omega_k a_\pi(\mathbf{k}) \tag{75}$$

$$[a_\pi(\mathbf{k}), H_I^\pi] = J_N(\mathbf{k}). \tag{76}$$

Consider now the perturbation expansion of the full $NN \to NN$ Green function with respect to the interactions $H_I^\pi(1)$ and $H_I^\pi(2)$:

$$\langle \mathbf{p}_1', \mathbf{p}_2' | \frac{1}{E^+ - H'} | \mathbf{p}_1, \mathbf{p}_2 \rangle = \langle \mathbf{p}_1', \mathbf{p}_2' | \frac{1}{E^+ - H_1 - H_2 - H_0^\pi} H_I^\pi(1) \frac{1}{E^+ - H_1 - H_2 - H_0^\pi} H_I^\pi(2) \frac{1}{E^+ - H_1 - H_2 - H_0^\pi} | \mathbf{p}_1, \mathbf{p}_2 \rangle + \ldots \tag{77}$$

where the term of order $H_I^\pi(1) H_I^\pi(2)$ has been singled out of the complete perturbation series, as it is just this term that coincides with the exact OPE potential specified by the original Hamiltonian H. Note how the introduction of the three Hamiltonians H_1, H_2, and H^π, enables us to treat meson exchange perturbatively, while nucleon dressing is treated non-perturbatively.

Now consider only this OPE term. Replacing $H_I^\pi(1)$ and $H_I^\pi(2)$ by the integral of Eq. (73), and then using Eq. (75), we obtain that

$$G_{12}^{OPE} = \int d\mathbf{k}\, \langle \mathbf{p}_1', \mathbf{p}_2' | \frac{1}{E^+ - H_1 - H_2} J_{N_1}^\dagger(\mathbf{k}) \frac{1}{E^+ - H_1 - H_2 - \omega_k} J_{N_2}(\mathbf{k}) \frac{1}{E^+ - H_1 - H_2} | \mathbf{p}_1, \mathbf{p}_2 \rangle \tag{78}$$

where we have used that H_0^π acting on two-nucleon states gives zero, and where one $\mathbf{k}'$-integral has been eliminated using $[a_\pi(\mathbf{k}'), a_\pi^\dagger(\mathbf{k})] = \delta(\mathbf{k}' - \mathbf{k})$. The essential step comes at this stage when we recognize that $[H_1, H_2] = [H_1, J_{N_2}] = [H_2, J_{N_1}] = 0$ which enables us to write Eq. (78) in terms of two contour integrals:

$$G_{12}^{OPE} = \left(-\frac{1}{2\pi i}\right)^2 \int d\mathbf{k}\, dz\, dz' \langle \mathbf{p}_1'| \frac{1}{z'^+ - H_1} J_{N_1}^\dagger(\mathbf{k}) \frac{1}{z^+ - H_1} |\mathbf{p}_1\rangle \frac{1}{z' - z - \omega_k} \langle \mathbf{p}_2'| \frac{1}{E^+ - z' - H_2} J_{N_2}(\mathbf{k}) \frac{1}{E^+ - z - H_2} |\mathbf{p}_2\rangle \tag{79}$$

where the matrix element factors into two, one factor for each nucleon. Because of this factorization, and because all Hamiltonians have the same form as H, we may now drop the nucleon labels in Eq. (79).

The matrix elements in Eq. (79) define the two-energy vertices

$$\delta(\mathbf{p}' + \mathbf{k} - \mathbf{p}) f(\mathbf{p}', \mathbf{p}, z', z) = g^{-1}(z', \mathbf{p}')\, \langle \mathbf{p}'| \frac{1}{z'^+ - H} J_N(\mathbf{k}) \frac{1}{z^+ - H} |\mathbf{p}\rangle\, g^{-1}(z, \mathbf{p}). \tag{80}$$

Substituting this definition into Eq. (79) results in the expression of Eq. (48).

Using Eqs. (75) and (76), it is straightforward to show that

$$\begin{aligned} &\langle \mathbf{p}'| \frac{1}{z'^+ - H} J_N(\mathbf{k}) \frac{1}{z^+ - H} |\mathbf{p}\rangle \\ &\quad = \langle \mathbf{p}'\mathbf{k}| \frac{1}{z^+ - H} |\mathbf{p}\rangle + (z - z' - \omega_k) \langle \mathbf{p}'| \frac{1}{z'^+ - H} a_\pi(\mathbf{k}) \frac{1}{z^+ - H} |\mathbf{p}\rangle\ . \end{aligned} \tag{81}$$

Recognizing that the first term on the rhs is the usual one-energy vertex function, Eq. (81) shows that the one- and two-energy vertex functions coincide for on-mass-shell pions.

SUMMARY

Attempts to formulate few-body equations for the πNN system have a long history. Yet all attempts from the very first bound state model of the 1960's to the sophisticated $NN - \pi NN$ models of the 1990's had one basic feature in common, each attempted to simplify the field theory in question by truncating to some maximum number of pions. Unfortunately, this truncation results in an inconsistent treatment of nucleon dressing with serious consequences for practical calculations. The $NN - \pi NN$ equations, for example, suffer from a renormalization problem which effectively reduces the strength of the input πNN vertices, which in turn contributes to the underestimation of cross sections.

In order to resolve these renormalization problems, we have presented a completely different approach where the Hilbert space is not truncated to some maximum number of pions. Instead, the guiding principle has been to work only with fully dressed vertices and propagators. What this means in practice, is that all possible disconnected diagrams of quantum field theory have been retained in our model. To achieve this, we have used convolution integrals to sum over all relative time orderings of disconnected graphs. As convolution integrals have often been used in nuclear physics, especially in the four-nucleon problem [21], it may appear surprising why the idea has not been applied sooner to the πNNproblem. This, however, can be understood when it is realized that the convolutions that we have used are applied to field theory Hamiltonians, and result in convolution expressions for *energy-dependent* "potentials". This is quite different from the more usual convolutions of nuclear physics which apply *only* for *energy-independent* potentials.

The convolution πNN equations are unitary and do not suffer from renormalization problems. These are the two crucial attributes that are necessary for a theoretically

consistent description. Indeed, we should emphasize that the downfall of the $NN-\pi NN$ equations is *not* because the two-nucleon propagator is underdressed, rather, it is simply because the equations themselves do not have both the properties of unitarity and correct renormalization.

The only approximation made in deriving the convolution πNN equations is the neglect of connected three-body forces. This is, of course, also an assumption in essentially every model in nuclear physics. Drawing on the bulk experience of the field, such three-body forces are very likely to be small. However, to obtain a more quantitative assessment, explicit calculations of connected three-body force contributions are needed. With this goal in mind, we have extended the convolution idea also to connected diagrams.

Two methods have been presented to derive convolution integrals that sum all the time-orderings of a connected graph. The first method, involving the two-time Green function of relativistic field theory, is useful to obtain the form of the convolution integral in a quick and straightforward manner. However, this method works only in the case when dressed and bare vaccua are equal. The second method, is an extension of the one used by us to derive the convolution formula for disconnected diagrams [13], and involves replacing the true Hamiltonian H by a sum of Hamiltonians, each having the same form as H, but applying only to singled out particles. This method works without any assumptions and is therefore more general than the one involving the two-time Green function.

With the convolution formula for connected diagrams derived, we have calculated the fully dressed NN OPE potential, since part of this potential includes connected three-body forces. For this dressed OPE potential, we find that the corresponding connected three-body force is negligible. Thus we can conclude, that at this stage, the convolution πNN model is a candidate for effectively summing the whole of time-ordered perturbation theory for the πNN system.

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